78-132

Application of DSN Spacecraft Tracking Technology to Experimental Gravitation

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Spacecraft tracking technology of the Deep Space Net (DSN) has been used in the past to measure the general relativistic increase in round trip group delay between the Earth and a spacecraft. As the DSN technology continues to improve, other gravitational experiments will become possible. Two possibilities are discussed in this paper. The first concerns the application of solar system dynamics to the testing of general relativity. The second involves the detection of VLF gravitational radiation (10^{-1} to 10^{-4} Hz) by means of the Doppler tracking of spacecraft.

Introduction

SPACECRAFT tracking technology of the Deep Space Net (DSN) has been used in the past to measure the increase in round trip delay, predicted by general relativity, for signals propagating between the Earth and a spacecraft near superior conjunction. The prediction has been verified to an accuracy of 3% with Mariner 6 and 7,[1] to 2% with Mariner 9,[2,3] and most recently to 0.5% with data from the Viking orbiters and landers.[4] Undoubtedly these experiments will continue in the future, and repeated measurements of the relativistic delay will be obtained to a fraction of a percent over a wide range of geometries. Such measurements will be effective in establishing confidence in the propagation result and also in testing another prediction of general relativity, that the speed of light is isotropic with respect to an inertial frame of reference.

However, as the DSN technology improves further, there will be a corresponding increase of interest in performing other gravitational experiments. Two possibilities are discussed in this paper. The first concerns the application of tracking data to solar system dynamics for purposes of testing the orbital predictions of general relativity. The second involves the detection of VLF gravitational radiation (10^{-1} to 10^{-4} Hz) by means of the Doppler tracking of spacecraft at round trip light times which are large ($>10^3$ s) compared to the duration of the gravitational wave pulses.

The solar system dynamics program will require accurate tracking of planetary orbiters and interplanetary spacecraft. Of all the space missions currently being considered for the 1980's, the Mercury Orbiter and Solar Probe seem the most favorable for this purpose and they are discussed here. State-of-the-art ranging, with accuracies on the order of 2 m, is also discussed from the viewpoint of what it can contribute to experimental gravitation on a Mercury orbiter mission. The possibilities for future improvements in ranging accuracies, perhaps to the 10-cm level, are considered. The basic conclusion is that future improvements will require the development of spacecraft ranging technology in the X-band region of the microwave spectrum.

The detection of VLF gravitational radiation depends on Doppler data. The current status of Doppler tracking accuracies can be compared with the expected signal level from sources of gravitational radiation.[5-7] On the basis of this comparison it is concluded that although it is worthwhile to conduct searches for gravitational radiation with current Doppler stabilities of $\pm 2\times 10^{-14}$ using hydrogen masers, it is more likely that a search will succeed with Doppler systems stable to $\pm 10^{-16}$. This will require new frequency standards, probably superconducting cavity stabilized oscillators (SCSO) and of course two-way coherent transmission. Also, because of plasma scintillations, it will be necessary to operate at X- and K-band frequencies in order to realize the full capabilities of the improved frequency standards. Real progress in experimental gravitation will be achieved only if the plasma problem can be eliminated. Dual-frequency uplink, for example, at S and X band, must be available for transmission to the spacecraft. The downlink can rely on the existing S- and X-band dual-frequency capability, but this must be supplemented by the addition of a K-band frequency for experiments very near the solar limb.

In the area of spacecraft systems, it will be necessary to implement drag-free control, or equivalent accelerometer systems, for the purpose of eliminating nongravitational forces to a level of at least $10^{-10}g$. Advances are also required in the spacecraft radio system. A program of transponder improvements is vital to experimental gravitation. The most important concerns include an X-band receive and K-band transmit capabilitiy, improved solid-state circuitry, broadband ranging with the capability to clean up the received modulation before transmission, and an improvement in phase stability, at least comparable to the ground-based frequency standards operating at one part in 10^{16}.

Doppler Tracking and Timing

For purposes of discussing the application of Doppler data to gravitational wave astronomy, it is instructive to consider a schematic representation of the response of two-way tracking to various physical influences (see Fig. 1). The three-pulse signature of a gravity wave and the relative amplitude of the pulses are shown in the bottom right diagram of Fig. 1. The rigorous derivation of this result, in terms of the angle of incidence θ of the incoming plane-transverse wave on the Earth-spacecraft line, can be found elsewhere.[5] The round trip light time, RTLT $=2l/c$, defines the low-frequency limit of the VLF band. Figure 1 shows the effect of clock jitter, plasma, and spacecraft buffeting on the Doppler signal. Each has characteristic signatures. A passing gravitational wave pulse has an effect that can be said to be a superposition of equal spacecraft and Earth buffeting, as well as an effect on clocks. Yet the principle of equivalence guarantees that there are no locally observable effects. As $l\to 0$, the three pulses of

Presented as Paper 78-132 at the 16th Aerospace Sciences Meeting, Huntsville, Ala., Jan. 16-18, 1978; submitted Feb. 13, 1978; revision received Sept. 5, 1978.

Index category: Spacecraft Signatures and Tracking.

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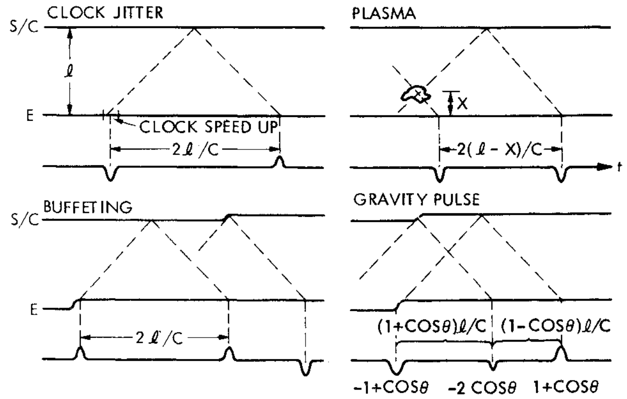

Fig. 1 Physical influences on two-way Doppler data.

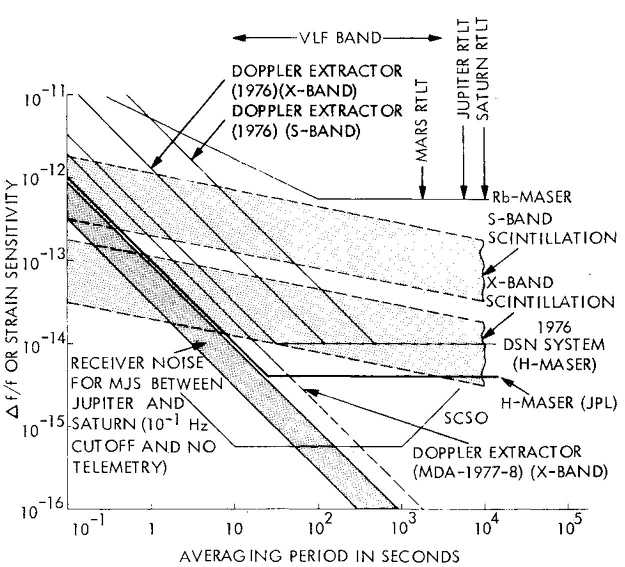

Fig. 2 Fractional frequency stability of various Doppler elements as a function of integration time.

the characteristic response blend and cancel. Consequently, the pulses must be short in duration compared to 2 l/c; otherwise, they will be undetectable. It is an advantage that the three-pulse nature of the gravitational wave is unique. An unambiguous detection is more likely, and it is easier to extract the gravitational signal from a Doppler record contaminated by the other noise sources of Fig. 1.

Engineering conservatism holds the guaranteed stability of the overall station timekeeping uncertainty to an rms figure $\sigma_y = 2\times 10^{-14}$, where $y = \Delta f/f$ is the instantaneous fractional frequency deviation, but it is perhaps not unreasonable to hope that a figure closer to that of the masers, 4×10^{-15}, might be achievable with care, for planned experiments. There are problems with phase delays, often temperature sensitive, in cables and transfer electronics, but various cable calibration and improvement tasks will take place as the DSN is methodically improved.

The VLF band for DSN observation of gravity waves has a low-frequency limit of perhaps 3×10^{-4} Hz set by the round-trip light travel time, 2 l/c, to missions to Jupiter and Saturn. The high-frequency limit is 5×10^{-2} Hz, set by the rise of hydrogen clock jitter for integration times < 20 s. Other clocks are worse. So this band does indeed seem to be an observational window. Looking through this window, we must learn both to discriminate the three-pulse signals from all the other competing influences, and to reduce these last to a minimum.

The fractional frequency-stability curve for the JPL designed hydrogen masers, which are now operational at all DSN 64-m stations, is shown in Fig. 2 along with other curves for the prior rubidium maser, the 1976 DSN Doppler system, with the hydrogen maser on line, and a prototypical SCSO.[27] Spacecraft receiver noise, S-band plasma scintillations, and X-band scintillations are shown as areas in the diagram. The capabilities of the current (1976) Doppler extractor and an improved extractor (MDA-1977-8), which will become operational soon, are also shown in Fig. 2.

To understand the Doppler readout precision, consider a single frequency ν_T being transmitted, multiplied in the spacecraft by a rational fraction K, coherently transponded, and then received back on the ground, Doppler shifted, as ν_R, so the Doppler frequency $K\nu_T - \nu_R$ is to be measured. Mainly to resolve ambiguity in observing very slowly moving spacecraft, a precisely known bias frequency, much lower than ν_R but usually large compared to the Doppler frequency, is also mixed in, so the Doppler readout electronics must measure a closely sinusoidal signal of frequency $K\nu_T - \nu_R + \nu_B$. The current bias frequency ν_B is 10^6 Hz. The precision of the system is achieved by counting the number of zero crossings, $n-1$, in a precisely known integration time T, and then determining the excess time after the last, τ, to a precision $\Delta\tau$. The new metric data assemblies (MDA) can resolve the excess time to 1.6×10^{-9} s. This translates to 0.55 deg out of each cycle. At X band this is an uncertainty of $1.3\times 10^{-13}/T$, which is only about a factor of 3 worse than the hydrogen maser fluctuation curve, as shown in Fig. 2. Operation with zero bias would more than achieve the goal of equaling the maser. The metric data assemblies will have four-channel capability at each 64-m DSN station, and hence they could read out simultaneously two S-band and two X-band Doppler records. We conclude that Doppler readout will not be a limiting problem for some time.

Of more concern, so far as system Doppler stability goes, is the transponder performance. Transponders are the one part of the system not under the direct control of the DSN. Present plans call for versions of the NASA Standard Transponder on future missions. This transponder does not include an X-band receive capability, and for this reason alone the standard transponder is inadequate for gravitational wave astronomy. Also, as the DSN system continues to improve, it will be important that the transponder phase stabilities be equal or superior to the rest of the system. Phase fluctuation spectra of the transponder in the VLF band must be obtained in the laboratory and be made available to the DSN as well as to the experimenters concerned with radio science on future missions. Such data are essential to an overall evaluation of phase stability in the two-way Doppler system.

Another important aspect of Fig. 2 is that the plasma noise at both S and X bands is larger than the hydrogen maser noise, and more than an order of magnitude larger than the SCSO. We must operate at X-band on both uplink and downlink, and have simultaneous S-band data available as well in order to remove the effects of plasma scintillations on the radio transmission. This is the key technological improvement needed, not only for future gravitational wave astronomy but also for high precision range tracking of the inner planets.

Ranging

The ranging system now employed for deep space missions switches the phase of an S-band carrier back and forth by a radian, say, every microsecond. This is equivalent to a 0.5-MHz square-wave phase modulation. There are, of course, other signals for command also put on the S-band carrier. The total bandwidth available is 10 MHz, but only about a third of this is for ranging. A microsecond corresponds to 300 m, one way, or 150-m range, and hence there is ambiguity modulo 150 m. This can be resolved by lower frequency modulation, or by much fancier pseudonoise pulse codes, in length up to a round trip light time, but in practice orbit determination predictions are sufficient, and one knows roughly the location of the spacecraft.

The current problem then is how well the electronic "ranging machines" can resolve the 0.5-MHz signal, and on knowing all the delays at antenna sites, in cabling, and in the spacecraft transponder. The official Viking Project requirements called for a 15-m ranging accuracy. For Voyager the requirement is considerably more stringent—4½-m range accuracy at Saturn. This is required because angular tracking (the other important precision input for spacecraft orbit determination) is read from the Earth rotation modulation of the Doppler, and Voyager at Saturn encounter happens to be at zero declination, precisely where this last fails. To achieve this improved ranging the DSN has sponsored a range-accuracy improvement effort that has pinpointed certain problem electronics, variable delays as the large antennas move about, spacecraft hardware calibration, and station calibrations. The result is that 1- to 2-m range accuracy has been achieved with the present DSN system. At least for the inner planets, any further improvement in S-band ranging is soon limited by solar plasma scintillation. Apparently, an even more basic limitation is in the bandwidth which could be improved for better resolution of the phase modulation. Both of these problems are alleviated by going to X-band uplink. A tenfold increase in ranging bandwidth would then be possible. Also all antenna gains pick up 12 dB. With X band, routine 50-cm ranging at Jupiter distance should be available within 10 years, and 10-cm ranging by 1990. However, improved ranging will be of little use if a free-fall orbit is not available for gravitational experiments. Each generation of spacecraft is getting more sophisticated, and this means that the unmodeled force or buffeting problem is constantly getting worse. With more stabilization and orienting capability, more on-board science, more gas jet action, the orbit is further away from pure free-fall. At perhaps 1-m range accuracy, a limit may already have been reached. The unmodeled force problem probably will be worse for Voyager than for Viking. So to really use 50 cm or less ranging accuracy for celestial science on a free spacecraft, drag-free systems, or equivalent accelerometers, are probably required. The other possibility, of course, is for experiments with planetary orbiters, also with reasonable care for insuring them to be inertial, or with planetary landers. Precision range tracking and celestial mechanics of the inner planets is a powerful means of investigating post-Newtonian predictions of relativistic gravity theories.

Very Long Baseline Interferometry

Present angle measurements are deduced from the Earth rotation modulation of Doppler data. This requires long intervals of Doppler tracking and is afflicted with plasma noise. Moreover, the scheme fails at low declinations. For Voyager navigation a significant new DSN capability will be tested, the application of very long baseline interferometry (VLBI). A spacecraft signal, modulated by an on-board noise source, is observed by two DSN stations with synchronized clocks. Accuracy of fractions of seconds of arc can be achieved, in short times. The allied technique of ΔVLBI finds the spacecraft angular position with respect to cosmic radio sources. Accuracies of about 10^{-2} arcsec should be available from VLBI in the 1980's, and ΔVLBI promises even greater accuracies, assuming of course that a radio source catalog of comparable accuracy is available. The entire question of VLBI and ΔVLBI technology for space navigation has been discussed.[8] Its potential for experimental gravitation has not been evaluated, and studies are needed in this area. However, current meridian circle observations of the inner planets yield angular positions accurate to about 1 arcsec, and it is not clear that the two orders of magnitude improvement offered by VLBI will be of great help. Radio metric range data are currently providing six orders of magnitude improvement over meridian circle observations, in an orthogonal direction of course, and it would seem that further improvements in ranging offer the highest yield for experimental gravitation. Yet, VLBI tracking of drag-free spacecraft is an intriguing possibility that deserves study.

Solar System Dynamics

For objects in the solar system, the first-order general relativistic effect in the observations is of order GM_s/c^2r, and at a characteristic distance of one astronomical unit.

$$\frac{GM_s}{c^2 r} \approx \frac{1.5\ \mathrm{km}}{1.5\times 10^8\ \mathrm{km}} = 10^{-8}$$

The subscript s refers to the Sun. If we want to test general relativity to an accuracy of 1% or better by means of solar system effects, then it is necessary to make measurements which are accurate to the order of one part in 10^{10} or better, although exceptions may arise. For example, the effects of general relativity on the propagation of signals which pass near the solar disk are of order GM_s/c^2R_s, or about one part in 10^6. One would think that if we are to have any hope of detecting second-order general relativistic effects ($\sim 10^{-12}$ at one R_s), an optimization of solar system experiments to take advantage of the stronger gravitational field very near the Sun is practically required.

The Schwarzschild solution of Einstein's equations for the one-body problem in isotropic coordinates is

$$\mathrm{d}s^2 = [(1-m/2r)/(1+m/2r)]^2 c^2 \mathrm{d}t^2 - (1+m/2r)^4(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2)$$

where

$$m = GM/c^2$$

To first order in m/r, this becomes

$$\mathrm{d}s^2 = [1-2(m/r)+2\beta(m/r)^2]c^2\mathrm{d}t^2 - [1+2\gamma(m/r)](\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2)$$

A Lagrangian $c\mathrm{d}s/\mathrm{d}t$ can be derived to the first order in m/r from $\mathrm{d}s^2$, and the equations of motion for a planet or spacecraft are as follows:

$$-\frac{c\,\mathrm{d}s}{\mathrm{d}t} = \frac{1}{2}v^2 + \frac{GM}{r} + \frac{1}{8}\frac{v^4}{c^2} - c^2 - \frac{1}{2}\frac{m}{r}\left(\frac{GM}{r} - 3v^2\right) + \ldots$$

This approach was used by Eddington,[9] with an added parameterization of the metric $\mathrm{d}s^2$, and was later used by Robertson[10] and Schiff[11] to analyze the physical significance of various tests of relativity. The metric for one body in this Eddington-Robertson-Schiff (ERS) formalism is

$$\mathrm{d}s^2 = [1-2(m/r)+2\beta(m/r)^2]c^2\mathrm{d}t^2 - [1+2\gamma(m/r)](\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2)$$

More recently, a very general parameterized post-Newtonian (PPN) metric has been introduced[12-15] which includes parameters (α_1, α_2, α_3) to account for possible preferred frame effects, and parameters (ζ_1, ζ_2, ζ_3, ζ_4) to account for a possible breakdown in the conservation of total momentum. A parameter ζ_w has been added by Will[15] to account for the possibility of galaxy-induced anisotropic effects in gravitational experiments. Modern descriptions of the dependence of various solar system experiments on these PPN parameters can be found in the literature.[12,16,17]

For the purpose of describing the post-Newtonian motions of the planets, it is sufficient to first write down the rigorous Newtonian equations of motion and then to add the various post-Newtonian accelerations for each planet. The one-body metric is the appropriate PPN metric for these additional accelerations. With an exception for the Earth-Moon system, which requires a few additional terms for orbit-orbit interactions planet-planet relativistic interactions can be neglected, at the level of current measurement accuracies.

In addition to the PPN effects, a number of other important parameters are included. These are, first of all, a parameter J_2 for the gravitational quadrupole moment in the Sun, secondly a parameter $\dot{G}/G$ for a possible time variation in the universal gravitational constant, and finally, for each body in the solar system, a parameter Δ_i which accounts for possible deviations from geodesic motion as considered by Nordtvedt.[18,19]

It is important to point out that although we express the motions of the planets in terms of the PPN formalism, this technique, at least from an experimental point of view, is largely a convenience. A planetary dynamics program whose goal is to test theories of gravitation must be a long-range effort extending over many decades. At the present time Einstein's theory of gravitation explains all radiometric data that have been accumulated on the planets and interplanetary spacecraft,[1-4] as well as all other modern data.[20-24] This is a remarkable fact, if one considers that we are using a theory that was developed when measurement accuracy was on the order of only 5×10^{-6}. Furthermore, general relativity has not broken down under the scrutiny of a new technology, developed 50 years after the introduction of the theory and now yielding accuracies of one part in 10^{11}. Thus our goal in analyzing the new radiometric data is not to determine values of the PPN parameters, simply as another set of physical constants, but instead to determine whether Einstein's theory will continue to remain valid for solar system gravitational experiments. In this sense the PPN formalism is very useful for characterizing a particular experiment and in quantifying results of fits to the planetary data, but it is important to keep in mind that subsequent analysis of solar system data over the next few years or decades might reveal inconsistencies with theory of a different nature than anything predicted by the PPN metric.

Time-Varying Gravitational Constant

It is possible that the gravitational constant G may be changing with time. We examine here the effect of such a change on radiometric data and show that the planetary data should be able to resolve whether or not G is affected by a cosmic distribution of mass as suggested by Brans and Dicke.[25]

The equation of motion of a planet with respect to the Sun with a time-varying value of G can be written

$$\ddot{r} = -\mu_0(1-Ht)(r/r^3)$$

and if H is equal to the value of the Hubble constant, it is about 5.6×10^{-11}/year.

Now because the time derivative of $r\times\dot{r}$ is zero, the orbital angular momentum h is conserved, and an integral of motion in polar coordinates is

$$r^2\dot{\theta} = h$$

and the equation of motion in polar coordinates is

$$\ddot{r} - r\dot{\theta}^2 = -\mu_0(1-Ht)/r^2$$

These two equations can be solved, in general, for r and θ as a function of time, but the solution for circular motion ($\dot{r}\equiv0$) will suffice. The solution to the first order in Ht is

$$r = r_0(1+Ht)$$

$$\dot{\theta} = \dot{\theta}_0(1-2Ht)$$

The first-order perturbation in the radial direction and along the orbit path is given, respectively, by

$$\Delta r = rHt$$

$$r\Delta\theta = -VHt^2$$

where V is the orbital velocity ($r\dot{\theta}$).

For coordinates of Mars with respect to the Earth the perturbation is

$$\delta(\Delta r) = (r_{\male} - r_{\oplus})Ht \sim 4.4\,t_{\rm yr} \quad \text{(m)}$$

$$\delta(r\Delta\theta) = (V_{\oplus} - V_{\male})Ht^2 \sim 10\,t_{\rm yr}^2 \quad \text{(m)}$$

No other dissipative effects of this magnitude have been suggested for planetary orbits, and it appears that a few years of Mars data should be able to resolve unambiguously whether or not G is changing on a scale comparable to the Hubble constant. This could be the most important result of the analysis of the Mariner 9 and Viking data, particularly if one recognizes that comparable results from the lunar motion will always be confused with geophysical dissipative effects.

A detailed covariance analysis for all the existing planetary data plus simulated Viking data has been carried out.[2] The results support the conclusion that $\dot{G}/G$ can be determined from one or two years of Viking data to an accuracy of at least $\pm5\times10^{-12}$. In addition, 2.5 or more years of Viking data can be used to determine the Nordtvedt deviations from geodesic motion to the same level of accuracy (±0.03) as the lunar laser result. The planetary result would be independent of the lunar result in the sense that the Nordtvedt effect on the lunar motion depends on the internal gravitational energy of the Earth,[18,19] while the effect on Mars depends on the internal gravitational energy of the Sun.

The results of the covariance analysis are plotted as a function of time in Figs. 3 and 4.

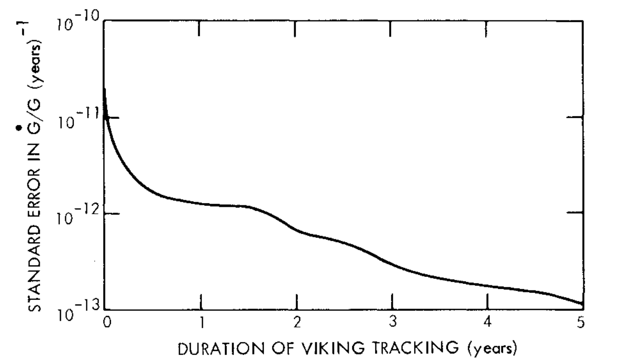

Fig. 3 Standard error on $\dot{G}/G$ as a function of time for simulated Viking range data.

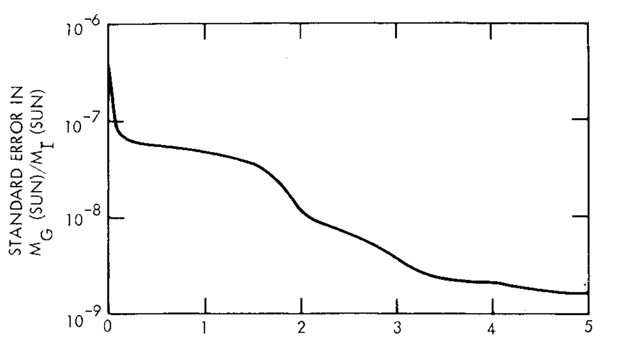

Fig. 4 Standard error on the ratio M_G/M_I of passive gravitational mass to inertial mass for the Sun as a function of time for simulated Viking range data.

Future Prospects

The analysis of Viking lander data is demonstrating that it is possible to fit the motion of Mars to the radiometric data with an rms range residual of about ± 2 m[4]. Continued experiments of this type will be very valuable to experimental gravitation in the future in that first-order general relativistic effects can be tested to an accuracy on the order of one part in 10^3. By taking advantage of long time intervals of several decades and by improving the accuracy of radiometric data by one or two orders of magnitude, it is conceivable that accuracies approaching one part in 10^6 or 10^7 of the first-order effects can be attained from the secular planetary terms. However, second-order general relativistic effects in the planetary motions will remain inaccessible for many decades to come, unless a new technology can be applied to the planetary data. Meanwhile radio propagation experiments and solar probes near the Sun should be relied upon for second-order effects.

A determination of the relativistic time delay, and hence the PPN parameter γ, to a fraction of a percent is possible with the Viking data. This represents the first opportunity to use dual-frequency ranging (S and X band) to reduce the contribution of the solar corona to the uncertainty in the measurement of γ. Later, an annual opportunity to perform a dual-frequency measurement will be provided by the Voyager mission to Jupiter and Saturn, and the JOP mission at Jupiter.

Very preliminary studies of a planetary orbiter about Mercury and of a solar probe in the vicinity of four solar radii of the Sun have been conducted.[26] Summaries of these studies are included in the next two sections.

Solar Probe

The mission uses Jupiter gravity assist to achieve a perihelion distance of about four solar radii. Time of flight to Jupiter is about 1.5 years, and the flight time from Jupiter to the Sun is about 2.0 years. The total mission duration of 3.5 years can be shortened by as much as one year for high-launch energies. With low launch energies of 110 km^2/s^2, the Space Transportation System can inject 850 kg. This is more than sufficient for a basic solar probe design.

The spacecraft bus is protected by a roof-type thermal shield for solar encounter. Made of MgO or $MgCO_3$ this shield will keep the bus temperature below 60°C for the duration of the flyby, and consequently, an extended mission is possible. A maneuver of 1.3 km/s at perihelion could be used to provide annual close flybys of the Sun.

A drag-free control system must be included with an accuracy of $10^{-10}g$ in all three axes of translational freedom. It is estimated that about 30 kg of propellant will be needed over the duration of the standard mission. Considerable analysis and modeling work is needed before the drag-free system can be specified.

Perhaps the most difficult technological problem is associated with the telecommunication system. Because of the high electronic interference near the Sun, the radio system will have a much lower signal-to-noise ratio than is typical of other space missions, and hence with the standard NASA transponder, the receivable bit rate will be lower ($\sim 10^3$ bits/s). In addition, the Doppler accuracy will be degraded significantly near perihelion, unless innovative radio techniques are used. A preliminary assessment of the problem suggests that an upgrading of the spacecraft transponder and DSN radiometric system will be required so that X-band (3 cm) uplink and downlink can be used during two-way tracking near the Sun. This possibility needs further study and analysis, as do a number of other options, including the incorporation of a hydrogen-maser frequency standard on board, and a transmission from the spacecraft in the K band.

The primary scientific data from a solar probe will be collected inside of 15 solar radii, over a time interval of about 33 h. The gravitational quadrupole moment J_2 of the Sun will be obtained from the Doppler tracking to an accuracy of $\pm 2\times 10^{-8}$. Current solar interior models predict a value of J_2 between 8.3×10^{-8} and 1.4×10^{-7}. Therefore, a measurement from solar probe will place fairly tight bounds on the range of acceptable solar models, and will show that some current models are not viable.

The tests of general relativity can be performed to better than 0.5% with Doppler tracking near the Sun. Improvements would result from the incorporation of an on-board hydrogen maser. Further studies are needed to define the expected accuracy of scientific measurements with the hydrogen maser.

The field and particle instruments are expected to provide in situ measurements of the intensity and pattern of the magnetic field inside of 10 solar radii, and of the density and velocities of electrons and nuclear particles. These data will lead to a better understanding of the generation of the interplanetary plasma. Considerable effort is required before the design of the particle and field instruments can be specified.

Mercury Orbiter

Range and Doppler data from the orbiter will be used to determine the orbit of Mercury over a period of at least one year. The technique consists of determining the motion of the center of mass of Mercury by tracking the orbiter. The determination of the orbit of the spacecraft with respect to the center of mass of Mercury is crucial, but a preliminary assessment of this determination indicates that the distance from the center of mass of Earth to the center of mass of Mercury can be achieved to an accuracy of ± 10 m with existing radio technology. Additional studies are needed to refine this result and to determine the impact of improvements in the DSN radiometric system on the determination of the Mercury orbit. Submeter accuracies are not out of the question.

The interest in a Mercury orbiter for gravitational experiments is high. The present difficulty with the Mercury orbit is that the best positional data are obtained by passive radar ranging from the surface. Mercury is not a particularly good radar target, and that, in combination with uncertainties contributed by the planetary topography, limit the radar distance determinations to Mercury to an accuracy of about ± 2 km. Two or three orders of magnitude improvement are feasible with an orbiter, and consequently, gravitational experiments can be performed to much higher precision than at present.

In order to have an idea of the relative merit of a Mercury orbiter in relation to other gravitational experiments, a preliminary error analysis has been performed, which although quite conservative and not complete, is nevertheless very illuminating.

The cursory covariance analysis of the Mercury orbiter problem has been performed under the following assumptions:

1) One measurement (normal point) to the center of mass of Mercury is available each day with a standard deviation of 10 m on each point.

2) Normal points are available on a daily basis over a time period of two years.

3) By means of dual-frequency tracking, both in range and Doppler, error contributions to the normal points from signal propagation effects in the solar corona are negligible.

4) Data are excluded from the covariance analysis when Mercury is less than 5 deg from the center of the Sun. This assumption is intended to account for possible difficulties with Doppler tracking of the orbiter when Mercury is near the Sun at superior conjunction. No points are excluded at inferior conjunction.

5) The parameters included in the covariance analysis are the six orbital elements of the Earth, assumed known initially

Table 1 Results of error analysis of the Mercury orbiter

Parameter:	β		γ		J_2	
Case	Initial error	Final error	Initial error	Final error	Initial error	Final error
1	5×10^{-2}	3×10^{-3}	2×10^{-2}	5.8×10^{-4}	5×10^{-5}	3.7×10^{-7}
2	10^{-3}	0.95×10^{-3}	10^{-4}	0.99×10^{-4}	5×10^{-5}	1.2×10^{-7}
3	4.5×10^{-3}	1.1×10^{-3}	10^{-2}	5.6×10^{-4}	10^{-8}	10^{-8}

to one part in 10^6, six orbital elements for Mercury, also assumed known initially to one part in 10^6, the relativity parameters (β, γ), the solar quadrupole, J_2, and the astronomical unit (AU), assumed known initially to 1 km.

A number of 16×16 covariance matrices were computed for various assumptions on the initial errors in β, γ, and J_2. The results are given in Table 1.

The basic conclusion of this cursory error analysis on the Mercury orbiter is that the chief benefit of this type of mission to experimental gravitation is in a very precise determination of the parameter γ. It is not possible to improve the parameter β very much beyond what can be achieved at present with lunar laser ranging data, and it would seem that several more years of lunar laser data will improve the accuracy of β to the point where the Mercury orbiter mission will not add very much additional information. With respect to the quadrupole moment, the Mercury orbiter would provide some new experimental data for the theoretical studies of the solar interior.

The present conclusions are very strongly dependent on the assumptions (1-5) of the Mercury orbiter error analysis. These assumptions are consistent with the radio science experience on Mariner 9 and the Viking orbiters at Mars, but it is quite possible that a Mercury orbiter mission, dedicated solely to general relativity and J_2 of the Sun, could be designed to yield better results with less tracking time. Also, future improvements in the DSN radiometric system will influence the expected results dramatically. Much more work is needed on studies and analysis of experimental gravitation on the Mercury orbiter before the real potential of this mission is understood.

Acknowledgments

This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, Calif., under Contract NAS7-100, sponsored by the National Aeronautics and Space Administration.

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